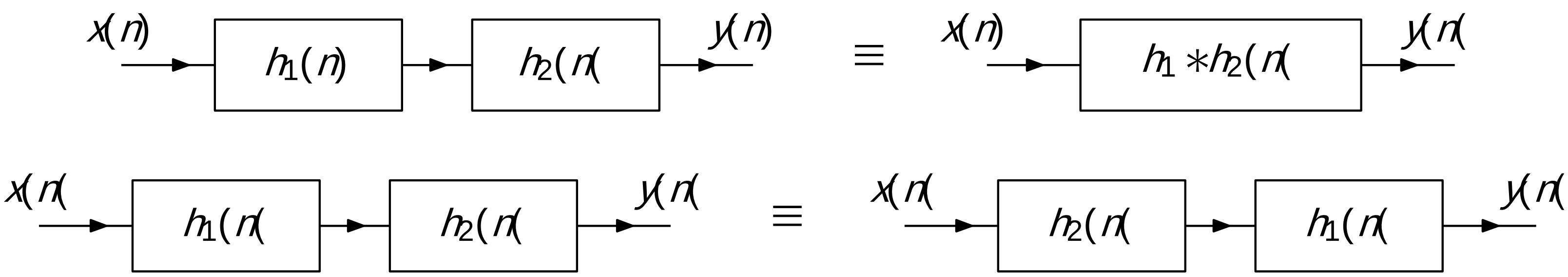
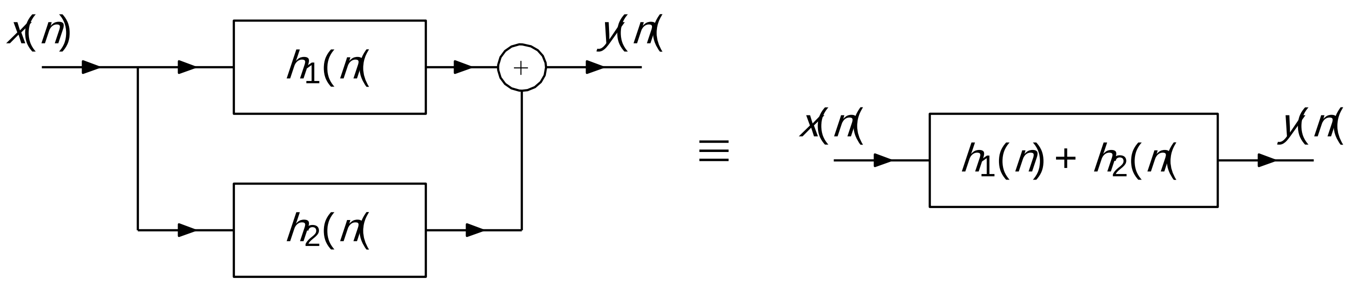
- The *series* interconnection of the LTI systems with impulse responses h_1 and h_2 is the LTI system with impulse response $h = h_1 * h_2$. That is, we have the equivalences shown below.



- The *parallel* interconnection of the LTI systems with impulse responses h_1 and h_2 is a LTI system with the impulse response $h = h_1 + h_2$. That is, we have the equivalence shown below.



Section 8.3

Properties of LTI Systems

- A LTI system with impulse response h is memoryless if and only if

$$h(n) = 0 \quad \text{for all } n \neq 0.$$

- That is, a LTI system is memoryless if and only if its impulse response h is of the form

$$h(n) = K\delta(n)$$

where K is a complex constant.

- Consequently, every memoryless LTI system with input x and output y is characterized by an equation of the form

$$y = x * (K\delta) = Kx$$

(i.e., the system is an ideal amplifier).

- For a LTI system, the memoryless constraint is extremely restrictive (as every memoryless LTI system is an ideal amplifier).

- A LTI system with impulse response h is causal if and only if

$$h(n) = 0 \quad \text{for all } n < 0$$

)i.e., h is a causal sequence.(

- It is due to the above relationship that we call a sequence x, satisfying

$$x(n) = 0 \quad \text{for all } n < 0,$$

a causal sequence.

- The inverse of a LTI system, if such a system exists, is a LTI system.
- Let h and h_{inv} denote the impulse responses of a LTI system and its (LTI) inverse, respectively. Then,

$$h * h_{\text{inv}} = \delta.$$

- Consequently, a LTI system with impulse response h is invertible if and only if there exists a sequence h_{inv} such that

$$h * h_{\text{inv}} = \delta.$$

- Except in simple cases, the above condition is often quite difficult to test.

- A LTI system with impulse response h is BIBO stable if and only if

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

(i.e., h is *absolutely summable*).

- An input x to a system $\mathcal{H}$ is said to be an eigensequence of the system $\mathcal{H}$ with the eigenvalue λ if the corresponding output y is of the form

$$y = \lambda x,$$

where λ is a complex constant.
- In other words, the system $\mathcal{H}$ acts as an ideal amplifier for each of its eigensequences x, where the amplifier gain is given by the corresponding eigenvalue λ.
- Different systems have different eigensequences.
- Of particular interest are the eigensequences of (DT) LTI systems.

- As it turns out, every complex exponential is an eigensequence of all LTI systems.
- For a LTI system H with impulse response h,

$$H\{z^n\} = H(z)z^n,$$

where z is a complex constant and

$$H(z) = \sum_{n=-\infty}^{\infty} h(n)z^{-n}.$$

- That is, z^n is an eigensequence of a LTI system and $H(z)$ is the corresponding eigenvalue.
- We refer to H as the system function (or transfer function) of the system H.
- From above, we can see that the response of a LTI system to a complex exponential is the same complex exponential multiplied by the complex factor $H(z)$

- Consider a LTI system with input x, output y, and system function H.
- Suppose that the input x can be expressed as the linear combination of complex exponentials

$$x(n) = \sum_k a_k z_k^n,$$

where the a_k and z_k are complex constants.
- Using the fact that complex exponentials are eigenfunctions of LTI systems, we can conclude

$$y(n) = \sum_k a_k H(z_k) z_k^n.$$

- Thus, if an input to a LTI system can be expressed as a linear combination of complex exponentials, the output can also be expressed as linear combination of the *same* complex exponentials.
- The above formula can be used to determine the output of a LTI system from its input in a way that does not require convolution.

Part 9

Discrete-Time Fourier Series (DTFS(

- The Fourier series is a representation for *periodic* sequences.
- With a Fourier series, a sequence is represented as a *linear combination of complex sinusoids*.
- The use of complex sinusoids is desirable due to their numerous attractive properties.
- Perhaps, most importantly, complex sinusoids are *eigensequences* of (DT) LTI systems.

Section 9.1

Fourier Series

- A set of periodic complex sinusoids is said to be harmonically related if there exists some constant $2\pi/N$ such that the fundamental frequency of each complex sinusoid is an integer multiple of $2\pi/N$.
- Consider the set of harmonically-related complex sinusoids given by

$$\phi_k(n) = e^{j(2\pi/N)kn} \quad \text{for all integer } k.$$

- In the above set $\{\phi_k\}$, only N elements are distinct, since

$$\phi_k = \phi_{k+N} \quad \text{for all integer } k.$$

- Since the fundamental frequency of each of the harmonically-related complex sinusoids is an integer multiple of $\frac{2\pi}{N}$, a linear combination of these complex sinusoids must be N-periodic.

- A periodic complex-valued sequence x with fundamental period N can be represented as a linear combination of harmonically-related complex sinusoids as

$$x(n) = \sum_{k=\langle N\rangle} a_k e^{j(2\pi/N)kn},$$

- where $\sum_{k=\langle N\rangle}$ denotes summation over any N consecutive integers (e.g., $0, 1, \ldots, N-1$). (The summation can be taken over any N consecutive integers, due to the N-periodic nature of x and $e^{j(2\pi/N)kn}$.)
- The above representation of x is known as the (DT) Fourier series and the a_k are called Fourier series coefficients.
- The above formula for x is often called the Fourier series synthesis equation.
- The terms in the summation for $k = K$ and $k = -K$ are called the Kth harmonic components, and have the fundamental frequency $K(2\pi/N)$. To denote that the sequence x has the Fourier series coefficient sequence a, we write

$$x(n) \overset{\text{DTFS}}{\longleftrightarrow} a_k.$$

- A periodic sequence x with fundamental period N has the Fourier series coefficient sequence a given by

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x(n) e^{-j(2\pi/N)kn}.$$

  (The summation can be taken over any N consecutive integers due to the N-periodic nature of x and $e^{-j(2\pi/N)kn}$.)
- The above equation for a_k is often referred to as the Fourier series analysis equation.
- Due to the N-periodic nature of x and $e^{-j(2\pi/N)kn}$, the sequence a is also N-periodic.